Application of the Simplified Models to Inverse Flood Routing in Upper Narew River (Poland)

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Abstract
In the paper a problem of inverse flood routing is considered. The study deals with Upper Narew River (Poland). To solve the inverse problem two approaches are applied, based on the kinematic wave equation and the storage equation, respectively. In the first approach, the hydrograph at the upstream end is determined via the inverse solution of the governing equation with backward integration in the $x$ direction. In the second approach, the standard initial value problem for the storage equation, completed by the steady flow equation, is solved with a negative time step, i.e., with an integration towards the diminishing time. It is shown that the proposed methods are equivalent.

Key words: open channel, flood wave, kinematic wave model, storage equation, inverse routing.

1. Introduction
Narew is a typical lowland river in the north-eastern part of Poland. The considered reach of length 100 km, between Siemianówka reservoir and gauge station Suraż, is surrounded by floodplains (Fig. 1.). This area comprises marshy meadows, famous for their exceptional biodiversity. It should be irrigated to ensure suitable conditions both for the wildlife species and for the farmers. To this order the water stored in the Siemianówka reservoir can be used. To ensure the requested flow conditions over the considered section of Upper Narew River, an appropriate reservoir operating strategy should be applied. Thus, we face a problem of flood control, typical in water management. The inverse flood routing appears to be a useful tool for decision makers responsible for water management in the valley of Narew River.

Usually the inverse routing is solved using the optimization technique. Sometimes it is solved via formulation of an inverse problem for the system of Saint Venant equations. For this reason let us recall the system of Saint Venant equations, the most
commonly used mathematical model of unsteady flow in an open channel. It can be written in the following conservation form:

\[ B \frac{\partial h}{\partial t} + \frac{\partial Q}{\partial x} = q, \]  
(1)

\[ \frac{\partial Q}{\partial t} + \frac{\partial}{\partial x} \left( \frac{Q^2}{A} \right) + gA \frac{\partial h}{\partial x} + gAS = 0, \]  
(2)

where: \( t = \) time, \( x = \) longitudinal coordinate, \( h = \) water stage, \( Q = \) flow discharge, \( A = \) cross-sectional area of flow, \( B = \) channel width at the water surface, \( g = \) gravitational acceleration, \( q = \) lateral inflow, \( S = \) slope of energy line expressed by the Manning formula.

Fig. 1. Map of the considered reach of Upper Narew River.

The presented system is usually integrated in a conventional way. It means that Eqs. (1) and (2) are solved over a channel reach of length \( L \) for increasing time \( t \), i.e., the integration is carried out in the following domain: \( 0 \leq x \leq L \) and \( t \geq 0 \) (Fig. 2.).

Let us consider the case of subcritical river flow, when \( \sqrt{gH} > U \) (\( H \) is flow depth and \( U \) is average cross-sectional velocity). Such a kind of flow is typical for the Narew River, because of its plane character. To ensure the uniqueness of solution, the following initial and boundary conditions must be imposed (Szymkiewicz 2000):

- initial conditions
  \[ Q(x,t) = Q_i(x) \text{ or } h(x,t) = h_i(x) \text{ for } t = 0 \text{ and } 0 \leq x \leq L; \]
boundary conditions
\[ Q(x,t) = Q_o(t) \quad \text{or} \quad h(x,t) = h_o(t) \quad \text{for} \quad x = 0 \quad \text{and} \quad t \geq 0, \]
\[ Q(t,x) = Q_L(t) \quad \text{or} \quad h(t,x) = h_L(t) \quad \text{for} \quad x = L \quad \text{and} \quad t \geq 0. \]

where \( Q(x), h(x), Q_o(t), Q_L(t), h_o(t) \) and \( h_L(t) \) are known functions imposed, respectively, at the channel ends. This so-called initial-boundary problem formulated for Eqs. (1) and (2) allows us to calculate the corresponding hydrographs at the downstream cross-sections for the flood wave imposed as the boundary condition at the upstream end of river reach.

![Direction of integration of the Saint Venant equations in the conventional approach.](image)

It is well known that the system of Eqs. (1) and (2) can be only solved numerically. To this order, the solution domain should be discretized and the differential equations should be approximated. The channel reach of length \( L \) is divided into \( M-1 \) intervals having length \( \Delta x \), while the time domain is divided with the time step \( \Delta t \). An approximation of the differential equations is carried out using the “box scheme”. This scheme works on grid point presented in Fig. 3.

All derivatives are approximated at point \( P \), which can move inside the mesh, with the following formulas (Cunge et al. 1980):

\[
\frac{\partial f}{\partial t}_P \approx \psi \frac{f^{i+1}_{j} - f^i_{j}}{\Delta t} + (1 - \psi) \frac{f^{i+1}_{i+1} - f^i_{i+1}}{\Delta t},
\]

\[
\frac{\partial f}{\partial x}_P \approx (1 - \theta) \frac{f^i_{j+1} - f^i_{j}}{\Delta x} + \theta \frac{f^{i+1}_{j+1} - f^{i+1}_{j}}{\Delta x},
\]

\[
f_p \approx \psi \left( \theta f^{i+1}_{i+1} + (1 - \theta) f^i_{i} \right) \left( 1 - \psi \right) \left( \theta f^{i+1}_{i} + (1 - \theta) f^i_{i+1} \right),
\]

where \( \psi \) and \( \theta \) are the weighting parameters ranging from 0 to 1.
Application of formulas (4), (5), and (6) to Eqs. (1) and (2) leads to a system of nonlinear algebraic equations, which must be completed by the imposed boundary conditions. This system is solved with an iterative method in each time step for marching time.

For the system of Saint Venant equations the so-called inverse problem can be formulated as well. On condition that the flow is subcritical, Eqs. (1) and (2) can be integrated over the assumed time interval \([0, T]\), in which the flow problem is analysed, towards diminishing \(x\) (Fig. 4.). Such approach can be applied when we want to determine the hydrograph at the upstream end that will ensure the required flow conditions at the downstream end.

The inverse problem for the Saint Venant equations may be solved either with the method of characteristics or with the method of finite difference, using the well-known four-point implicit scheme (box scheme), commonly applied for conventional
solution of these equations. The first approach was proposed by Bodley and Wylie (1978), whereas the second one was applied by Szymkiewicz (1993, 1996). However, the application of these approaches to real-life problems is not easy, especially when the rivers’ cross-sections have complex forms, as is the case for the river surrounded by a flood plain. In addition, the final system of algebraic equations for an inverse problem is much larger than for the direct problem. It is because in each river cross-section the governing equations are solved over the total considered time interval [0, T]. Since the acceptable time steps are relatively small, very large systems result.

It seems that the inverse flood routing can be carried out using simplified approaches. First of all, to model the unsteady flow, one can apply the simplified system of the Saint Venant equations in the form of the kinematic wave model, which is integrated backwards with respect to \( x \), as shown in Fig. 3. The advantage of such an approach is that the final system of non-linear algebraic equations can be split into separate non-linear equations which are solved subsequently using one of the standard methods.

The second approach is based on the storage equation and the steady-state flow equation. In such a way, instead of solving the inverse problem, the initial-value problem for the system of ordinary differential non-linear equations, representing the cascade of \( N \) reservoirs must be solved. This system is solved via the numerical integration over time with negative time step. It means that to determine the hydrograph at the upstream end, instead of solving the complex inverse problem for hyperbolic equations, one can solve a relatively simple initial-value problem.

In the paper both approaches to solve the inverse routing were applied. They are shown to be equivalent for certain conditions, related to the applied numerical methods.

2. Inverse flood routing using the kinematic wave model

The kinematic wave model is derived from the system of Saint Venant equations by neglecting the inertia and pressure forces in the momentum Eq. (2) (Cunge et al. 1980). It takes the following form:

\[
\frac{\partial h}{\partial t} + \frac{1}{B} \frac{\partial Q}{\partial x} = \frac{q}{B},
\]

\[
Q = \frac{1}{n} R^{2/3} S^{1/2} A,
\]

where \( s \) is the channel bottom slope, considered as an average for whole river reach of length \( L \). Let us approximate the continuity equation (7) using formulas (4), (5) and (6), previously presented for box scheme. One obtains:

\[
(1 - \psi) \frac{h_i^{j+1} - h_i^j}{\Delta t} + \psi \frac{h_{i+1}^{j+1} - h_{i+1}^j}{\Delta t} + \frac{1}{B_p} \left( (1 - \theta) \frac{Q_i^{j+1} - Q_i^j}{\Delta x} + \theta \frac{Q_{i+1}^{j+1} - Q_{i+1}^j}{\Delta x} \right) = \frac{q_p}{B_p},
\]

where \( i \) is the index of cross-section and \( j \) the index of time level.
\[ q_p = \psi \left( \theta q_i^{j+1} + (1 - \theta) q_i^j \right) + (1 - \psi) \left( \theta q_i^{j+1} + (1 - \theta) q_i^j \right), \quad \text{(10)} \]

\[ B_p = \psi \left( \theta B_i^{j+1} + (1 - \theta) B_i^j \right) + (1 - \psi) \left( \theta B_i^{j+1} + (1 - \theta) B_i^j \right), \quad \text{(11)} \]

\[ Q_i^j = \frac{1}{n} s_i^{j+1} (R_i^j)^{2/3} A_i^j. \quad \text{(12)} \]

In the case of conventional solution for the kinematic wave equations, the nodal values \( h_i^j \) and \( Q_i^j \) are known either from the initial condition or from the previous time step, whereas \( h_i^j (j=1,2,3,\ldots) \) are known from the boundary condition imposed at \( x=0 \). Consequently, in Eq. (8) only one unknown \( h_i^{j+1} \) exists. It can be calculated for the subsequent nodes along the \( x \) axis, proceeding from left to right, i.e., for \( i=2,3,4,\ldots,N \).

![Fig. 5. Solution domain and additional condition required by the kinematic wave model applied to inverse flood routing.](image)

Equations (8) and (9) can be used for inverse flood routing as well. For this problem, the following initial and boundary conditions should be imposed at the boundaries of the solution domain (Fig. 5):

- **initial conditions:**
  \[ Q(x,t) = Q_i^j \] and \( h(x,t) = h_i^j \) for \( x = L \) and \( 0 \leq t \leq T \);

- **boundary conditions:**
  \[ Q(t,x) = Q_i^j \] and \( h(t,x) = h_i^j \) for \( t = T \) and \( 0 \leq x \leq L \).

In this case the nodal values \( h_i^j \) and \( Q_i^j \) are known from the required hydrograph at \( x=L(i=N) \) or as a result of calculations carried out for the previous cross-section. In addition, \( h_i^j (j=1,2,3,\ldots) \) is known as the boundary condition expected at the moment \( t = T \). Therefore, similarly to the case of the direct solution, in Eq. (8) only one unknown \( h_i^j \) exists. It can be calculated for the subsequent nodes along the \( t \) axis following from the last time level \( t = T \) towards \( t = 0 \), i.e., for \( j = M-1, M-2, M-3, \ldots, 1 \).
(\(M\) is the total number of time steps). Since Eq. (8) is a function of \(h_i^j\) and contains a single root, it can be solved with the bisection method.

In the Manning formula (12) \(s\) is considered as the average longitudinal slope of the river bed and is assumed to be constant, whereas the cross-sectional area \(A(h)\) for compound channels with flood plain is replaced by the area of the active part of cross-section, i.e., by part of total cross-sectional area in which the main stream of flowing water takes part (Fig. 6).

Finally, the calculations carried out in the presented way allow us to determine the required hydrograph at the upstream end of considered channel reach.

3. Inverse flood routing using the storage equation

Let us consider a channel reach of length \(L\). Using \(n\) known cross-sections it can be divided into \(N–1\) intervals of length \(\Delta x = x_{i+1} - x_i\) \((i = 1, 2, \ldots, N–1)\). In such a way, one obtains a cascade of \(N–1\) reservoirs so that the outflow from the preceding reservoir is the inflow for the next one (Fig. 7).
The storage equation can be derived from the differential continuity equation (1). To this order, it is integrated with regard to \( x \) over a channel’s increment of length \( \Delta x \). Consequently, one obtains:

\[
\frac{dV_i}{dt} = Q_i - Q_{i+1} + I_i,
\]

(13)

where: \( i \) = index of reservoir and cross-section, \( V_i \) = total volume of water stored by channel reach, \( Q_i \) = flow discharge through cross-section \( i \), \( I_i \) = total lateral inflow into reservoir \( (= \Delta x \cdot q) \).

To calculate the flow discharge between the neighbouring reservoirs, the Manning formula (8) is used. As previously, in this equation \( s \), considered as average longitudinal slope of bed river, is assumed to be constant, whereas the cross-sectional area \( A(h) \) for compound channels with food plain is replaced by the area of the active part of cross-section.

After eliminating \( V \), Eq. (13) takes the following form:

\[
\frac{d\bar{h}_i}{dt} = \frac{1}{f_i}(Q_i - Q_{i+1} + I_i),
\]

(14)

where \( f_i \) is the area of reservoir’s surface (a function of the water stage).

The mean level of the water surface above the datum \( \bar{h} \) can be expressed using nodal values of \( h \) as follows:

\[
\bar{h} = \psi h_i + (1 - \psi)h_{i+1},
\]

(15)

where \( \psi \) is the weighting parameter ranging from 0 to 1. Finally, Eq. (13) takes the following form:

\[
\psi \frac{dh_i}{dt} + (1 - \psi) \frac{dh_{i+1}}{dt} = \frac{1}{f_i}(Q_i - Q_{i+1} + I_i).
\]

(16)

Similar equations can be written for every reservoir (for \( i = 1, 2, \ldots, N-1 \)). Consequently, a system of \( N-1 \) ordinary differential equations is obtained. Its solution is formulated as an initial-value problem. Knowing the initial condition \( h_i(t = 0) = h_{i,0} \), where \( h_{i,0}(i = 1, 2, \ldots, N-1) \) is given, the functions \( h_i(t) \) have to be calculated for \( 0 \leq t \leq T \). In this order, the equation should be numerically integrated using for example the following general two-level scheme:

\[
y_{j+1} = y_j + \Delta t \left( \theta y'_j + (1 - \theta) y'_{j+1} \right),
\]

(17)

where: \( j \) = index of time level, \( \theta \) = weighting parameter ranging from 0 to 1.

The value of the weighting parameter \( \theta \) defines the method of integration. For \( \theta = 0 \) one obtains the explicit Euler method, for \( \theta = 1/2 \) – the implicit trapezoidal rule and for \( \theta = 1 \) – the implicit Euler method (Press et al. 1992).

Application of Eq. (17) to Eq. (16) leads to the following formula:
for \( i = 1, 2, ..., N-1 \).

While solving conventional flood routing, the function \( h_0(t) \) is known as the flood wave imposed at the upstream end of channel. Therefore, Eq. (18) applied for the first reservoir \((i=1)\) contains one unknown only. Similar situation arises for the next reservoirs, since the outflow from the preceding one is the inflow to the next one. It means that system of equations (18) can be split and each equation can be solved separately with regard to the unknown \( h_{i+1}^{j+1} \):

\[
h_{i+1}^{j+1} = h_i^j - \frac{\psi}{1-\psi} \left( h_{i+1}^{j+1} - h_i^j \right) + \frac{1-\theta}{f_i^j} \Delta t \left( Q_i^j - Q_{i+1}^{j+1} + I_i^j \right) + \frac{\theta}{f_i^j} \Delta t \left( Q_i^{j+1} - Q_{i+1}^{j+1} + I_i^{j+1} \right)
\]  

(19)

for \( i = 1, 2, ..., N-1 \).

This equation is non-linear, since according to the Manning formula (12), \( Q_i^{j+1} \) depends on \( h_{i+1}^{j+1} \). To solve Eq. (19), an iterative method should be applied. Finally the calculations are going for increasing time, similarly to the situation presented in Fig. 2.

Fig. 8. Domain of integration of the storage equation and the required additional information for inverse routing.

Equation (18) can be used for the inverse flood routing. To this order a proper initial value problem should be formulated. One can assume that at the end of the considered time period \( T \) the steady flow was reached. It means that the condition \( h_i(t=T) = h_{i,\text{fin}} \) for \( i=1,2, ..., N \) is given. Moreover at the downstream end (cross-section \( N \)), the required hydrograph \( h_N(t) \) is imposed (Fig. 8). Now, integrating
Eq. (18) with a negative time step $\Delta t$ (see Fig. 9) one can arrive at the hydrograph at $x = 0$, i.e., to $h_0(t)$.

\[
\psi_{i+1} = h_{i+1} - \frac{1}{\psi}(h_{i+1} - h_i) + \frac{1-\theta}{\psi} \frac{\Delta t}{f_i} (Q_i - Q_{i+1} + I_i) + \frac{\theta}{\psi} \frac{\Delta t}{f_{i+1}} (Q_{i+1} - Q_{i+1} + I_{i+1})
\]

(20)

for $i = N-1, N-2, ..., 1$. This equation is a non-linear one. Its solution in the backward direction allows us to obtain at the upstream end ($x = 0$) the hydrograph corresponding to the one imposed at the downstream end ($x = L$).

4. Discussion of the numerical results

The two presented approaches were used to compute the flow in the Upper Narew River. Each one was applied for both direct and inverse problems. All the data required for the calculations had been provided by the Institute of Geophysics of the Polish Academy of Sciences.

The considered river reach between Siemianówka Reservoir and the gauge station Suraž has the length $L = 101.300$ km. Along the river course, $N = 54$ cross-sections were measured. The distances between the neighbouring sections vary from 0.360 km to 3.530 km. In Fig. 10 the shape of the cross-section at 27.920 km from Siemianówka Reservoir is shown as an example. For the considered reach of Narew River the presence of flood plains is typical. The river bed has the width of about 75 m whereas the width of the flood plains exceeds 1000 m.
For testing, the following general assumptions were adopted:

- In all cross-sections the active part is not separated;
- The lateral inflow is neglected;
- The initial condition is calculated via the solution of the equation for steady non-uniform flow with the discharge and water stage imposed at the downstream end;
- The condition imposed at the upstream and for the direct solution has the form of hydrograph prescribed by the following formulas:

  \[ Q(t) = Q_{in} \]

  for \( t \leq t_0 \)

  \[ Q(t) = Q_{in} + (Q_{max} - Q_{in}) \left( \frac{t}{t_{max}} \right)^{\alpha} \exp\left(1 - \left( \frac{t}{t_{max}} \right)^{\alpha} \right) \]

  for \( t > t_0 \)

  \[ (21) \]

  where \( Q_{in} \) is the initial discharge, \( Q_{max} \) is the peak discharge, \( t_{max} \) is time to peak, \( t_0 \) is lag time and \( \alpha \) is parameter;

- The condition imposed at the downstream end for the inverse routing has the form of hydrograph \( Q_L(t) \) which is given numerically;
- All non-linear algebraic equations are solved using the bisection method. For a function having a single root this method is always convergent (Press et al. 1992).

The results for the direct problem are shown in Fig. 11. For the same flood wave imposed at \( x = 0 \) (Eq. (21) with \( t_0 = 24 \) h, \( Q_{in} = 13.55 \) m\(^3\)/s, \( Q_{max} = 80 \) m\(^3\)/s, \( t_{max} = 36 \) h and \( \alpha = 1.25 \)), the hydrographs calculated at the downstream end (in Suraż) appeared...
practically identical. They were provided by the same set of weighting parameters, namely by $\theta = 1$ and $\psi = 0$. For the values of time step $\Delta t$ taken from 0.02 h to 0.2 h the results of calculations varied slightly. The stable solution was ensured on condition that $0.5 \leq \theta \leq 1$ and $0.5 \geq \psi \geq 0$.

Fig. 11. Results of flood wave routing (a – imposed at upstream end, b – calculated with the kinematic wave equation, c – calculated with the storage equation).

The seemingly surprising agreement of the results can be interpreted easily. Let us compare Eq. (9), being an approximation of the kinematic wave equation using the box scheme, and Eq. (18), which was obtained by solution of the storage Eq. (16) with formula (17). In fact, the two equations are identical, although derived in different manners. To confirm this identity let us multiply and divide the right side of Eq. (9) by $\Delta x$:

$$\frac{q_p}{B_p} = \frac{q_p \cdot \Delta x}{B_p \cdot \Delta x} = \frac{I}{B_p \cdot \Delta x}.$$

On the other hand, the area of the reservoir at the level of its surface in Eq. (18) can be expressed by the following approximating formula:

$$f \approx B_p \cdot \Delta x.$$

After introduction of the above relations into Eq. (9) and Eq. (18), respectively, they become identical formulas. This result is reasonable since both approaches have the same roots: the continuity equation and the Manning formula.

The only difference between both equations is that Eq. (9) represents an integration of the kinematic wave towards diminishing $x$ (see Fig. 4), whereas Eq. (18) expresses the numerical solution of the storage equation by the method (17) with negative time step towards diminishing time. As shown in Fig. 11, both approaches provide identical results.

The next example concerns inverse routing. At the downstream end (in Suraż) the hydrograph $Q_i(t)$, which has been previously computed via the direct solution, was imposed as the boundary condition. In this case, both approaches also gave very simi-
lar results. It appeared that stable solution is ensured for $0 \leq \theta \leq 0.5$ and $1 \geq \psi \geq 0.5$. These conditions are inverse to the ones for the conventional solution. This property of the box scheme applied to the inverse solution of the system of Saint Venant equations was shown by Szymkiewicz (1996). It holds for the kinematic wave as well, which is of hyperbolic type as the Saint Venant equations.

In Fig. 12 one can observe some oscillations of the hydrograph calculated at the upstream end (Siemianówka). They depend on the values of the weighting parameters and are caused by numerical dispersion. The displayed results were obtained for $\theta = 0.5$ and $\psi = 0.62$ (smooth curve) and for $\theta = 0.4$ and $\psi = 0.525$ (oscillating curve). The oscillations disappear with increasing numerical diffusion generated by the box scheme. It is controlled by $\theta$ and $\psi$. For $\theta = 0$ and $\psi = 1$ the numerical solution becomes smooth, whereas for $\theta = 0.5$ and $\psi = 0.5$ the oscillations are the most pronounced, since for these values the box scheme is dissipation-free. However, in such a situation it is practically impossible to obtain the effect of smoothing and at the same time to ensure an increase of steepness of the hydrograph at upstream end. In this case both approaches produce very similar solutions. In addition, they fulfil perfectly the law of mass conservation.

![Fig. 12. Results of inverse flood wave routing carried out with kinematic wave equation (a – imposed at downstream end; calculated at upstream end: b – for $\theta = 0.4$ and $\psi = 0.525$, c – for $\theta = 0.5$ and $\psi = 0.62$).](image)

Presented routing of the hypothetical flood wave allowed us to recognize general properties of the proposed methods of solution. In the next example, the observations of the discharges $Q(t)$, carried out at the gauges stations Bondary and Suraż between 07.03.1994 and 04.07 1994 were used. Comparison of both curves presented in Fig. 13 suggests that at the considered river reach the lateral inflow coming from subcatchments plays an essential role. Its estimation was based on the aforementioned observations as well as the observations made at the intermediate gauge stations (Narew and Ploski – see Fig. 1). Imposing as the boundary condition the hydrograph $Q(t)$ observed at the upstream end and the lateral hydrographs previously evaluated, the direct routing was
Fig. 13. Results of flood wave routing between Bondary and Siemianówka gauge station for the period 07.03 – 04.07.1994 (a – imposed at upstream end, b – observed at downstream end, c – calculated with storage equation).

Fig. 14. Results of the inverse flood wave routing between Bondary and Siemianówka gauge station for the period 07.03 – 04.07.1994 (a – imposed at the downstream end, b – calculated with the storage equation, c – observed).
performed using the storage equation. The results of calculations seem to be quite reasonable and similar to the results of the data provided by the experiment. They were obtained for $\Delta t = 0.25 \text{ h}$, $\theta = 1.0$, and $\psi = 0$. In the next numerical experiment, the routing process has been inverted. Using the same model and imposing the hydrograph at the downstream end as well as the lateral ones, the hydrograph at the upstream end was calculated. The results presented in Fig. 14 were given by $\Delta t = 0.50 \text{ h}$, $\theta = 0$, and $\psi = 1.0$. Also in this case they can be considered as reasonably accurate.

However, to better evaluate the applied model more data are needed. Unfortunately, at the moment they are not available.

5. Conclusions

Two alternative approaches were proposed to solve the problem of inverse flood routing for Upper Narew River, between the cross-sections Bondary and Suraż. Both are based on the simplified forms of the Saint-Venant system, i.e., the kinematic wave equation and the storage equation. Consequently, one avoids the solution of a relatively complicated problem leading to large systems of algebraic equations. Instead, in the case of the kinematic wave equation one has to solve a non-linear algebraic equation for each node separately. In the case of the storage equation an ordinary differential equation is integrated backwards in time, i.e. with the negative time step. It was shown that both approaches are equivalent and they ensure very similar results. The preliminary results presented in this work seem encouraging. However, in order to better evaluate the proposed approaches more hydrological data should be acquired from field experiments, especially concerning the lateral inflow from the subcatchments of Upper Narew River.

Acknowledgments. The research was supported by the Polish Committee for Scientific Research in the framework of the Project no 2 p04d 009 29.

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