The Impact of Hydraulic Conductivity on Topography Driven Groundwater Flow

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Abstract
Landscape topography is the most important driving force for groundwater flow and all scales of topography contribute to groundwater movement. Here we present results of how different scales of topography affect the groundwater flow at different depths. The study is based on a spectral analysis of the topography and a couple of exact 3-D solutions of the groundwater flow. We are also analyzing how different heterogeneities of the subsurface hydraulic conductivity impact the groundwater flow at different depths and alter the relative importance of different topographic scales on the groundwater flow. Quaternary deposits are extremely important for the infiltration at the ground surface, but the effect is primarily constrained to the deposit strata. Depth dependent hydraulic conductivity has a major impact on the size and depth of the groundwater flow cells, but it also affects the infiltration at the surface. Depth dependent hydraulic conductivity tends to counteract the effect of the large-scale topography on the groundwater flow more effectively than the smaller landscape scales.

1. Introduction
Freshwater is the most important natural resource for human life and, even if surface water reservoirs are refilled much more rapidly than groundwater reservoirs, groundwater (GW) constitutes 98% of all non-frozen freshwater (Shiklomanov 1998) and are therefore an important source for freshwater in many countries (i.e. Hutson et al. 2004). Due to the increasing demand of freshwater that follows from the world's rapid population growth there is an ever greater need for a better understanding of the processes controlling the groundwater recharge. Recharge of new groundwater is perhaps the most important factor for sustainable water resource management.

In humid climates, where the groundwater surface tends to follow the ground surface, differences in hydraulic potential created by topography are the main driving force for groundwater flow. The impact of different scales of topography on groundwater flow has been investigated for different reasons (i.e. Alley et al. 2002, Zilj
Recently we developed a method to represent the topography in three dimensions by a spectrum of harmonic functions (Wörman et al. 2006). With this approach we can estimate the relative impact of a certain scale of topography on groundwater movement at a specified depth.

In this study we have developed new three-dimensional, exact solutions to investigate how the scale effect of topography changes when more realistic representations of the subsurface are induced, especially depth dependent hydraulic conductivity and quaternary deposits on top of bedrock. By doing so, our aim is to give a better understanding of the relationship between topography and groundwater flow. This understanding is important, not only for water resource management, but also for many technical implementations; drainage in tunnel constructions, the performance of nuclear waste repositories (Marklund et al. 2007) etc.

In this study we investigate to what degree various scales of topography are controlling the recharge and, especially, how different depth-dependent conductivities as well as presence of Quaternary deposits control infiltration. For various representations of the subsurface, we study how different scales of topography control the groundwater movement at different depths.

2. Methods

Groundwater re- and discharges are controlled by landscape topography because the groundwater surface tends to follow the ground surface. We characterize the effects of the land surface topography on subsurface flow in terms of the vertical flux. This is done by performing a spectral analysis of the topography (Wörman et al. 2006) in three domains with different sizes and resolutions (Table 1).

<table>
<thead>
<tr>
<th>Name</th>
<th>Size</th>
<th>Resolution</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>350×350 km</td>
<td>141×141</td>
</tr>
<tr>
<td>M</td>
<td>100×100 km</td>
<td>201×201</td>
</tr>
<tr>
<td>S</td>
<td>30×30 km</td>
<td>241×241</td>
</tr>
</tbody>
</table>

All three domains are located in Sweden (Fig. 1) and the largest domain surrounds the middle-sized domain which is surrounding the smallest domain.

The result of the spectral analysis is a representation of the ground surface topography in a Fourier-series spectrum. Assuming a groundwater surface that follows the ground surface gives us the hydraulic potential as a boundary condition. Hence, the Fourier-series spectrum combined with the hydraulic conductivity provides an exact solution of the underlying three-dimensional groundwater flows induced by these topographic features over a wide range of spatial scales. According to Wörman and colleagues (2006), the solution to the vertical groundwater velocity component (vertical flux) in a homogeneous subsurface becomes:
\[ w(x, y, z) = \sum_{i=1}^{n_x} \sum_{j=1}^{n_y} (h_m)_{i,j} K \frac{\exp \left( \sqrt{k_{x,i}^2 + k_{y,j}^2} \right) + \exp \left( \sqrt{k_{x,i}^2 + k_{y,j}^2} (-2z - z) \right)}{1 + \exp \left( -2 \sqrt{k_{x,i}^2 + k_{y,j}^2} \right)} \cdot \sqrt{k_{x,i}^2 + k_{y,j}^2} \sin(k_{x,i} x) \cos(k_{y,j} y) \] (1)

in which \( h_m = \) amplitude coefficients [m], \( h = \) hydraulic head [m], \(< \ldots \> = \) arithmetic average value, \( N = \) number of wavelengths in the \( x \)- and \( y \)-directions, \( z [m] = \) the depth to a no-flow condition boundary, \( K = \) hydraulic conductivity and \((x, y, z) = \) Cartesian coordinates.

Fig. 1. Map of Sweden showing the three different study-domains: L, M and S.

2.1 Impact of depth dependent hydraulic conductivity

To quantify the groundwater recharge we integrate the absolute value of the vertical velocities over the entire domain area. This was performed in all three domains and at three different depths; \( z = 0, z = -500 \) and \( z = -1000 \) meters. The vertical flux at the different depths is calculated for two different representations of the hydraulic conductivity \((K)\). First we use a homogeneous conductivity, \( K = 10^{-6} \) m/s. The second representation is a depth dependent function described by Eq. 2.

\[ K = K_0 e^z \] (2)

where \( K_0 \) is the conductivity at the surface, \( z \) is the depth and \( c \) is a positive constant. For such hydraulic conductivity we have derived an exact solution for the vertical flow velocity:
By comparing Eqs. (1) and (3), we can evaluate how the depth dependent hydraulic conductivity affects the groundwater movement at certain depths and for different scales of topography. We based the values of $c$ and $K_0$ on borehole data from Sweden ranging down to approximately 1600 m depth (Rhen et al. 2006), $c = 0.00641$ and $K_0 = 1.925 \times 10^{-7}$.

2.2 Groundwater recharge through soil-rock interface

To study the effect of Quaternary deposits we have derived an exact solution for the groundwater flow with a layered representation of the subsurface consisting of two layers; the Quaternary deposits and the bedrock. Here we study how different thicknesses and different conductivities of the Quaternary deposits affect the flux at the interface between soil and rock. Since the analytical method (Eq. 4) we use here can only cope with soil layers of constant thickness, we have chosen to study the two different thicknesses: 2 and 10 m. We also study two different soil types, till ($K \sim 10^{-6}$) which is the dominating soil type in Sweden and sandy sediments ($K \sim 10^{-5}$) (Domenico and Schwartz 1998). The vertical velocity in the Quaternary deposits ($w_1$) and in the bedrock ($w_2$), are given in the following expression:

$$w(x, y, z) = \sum_{j=1}^{N_y} \sum_{i=1}^{N_x} (h_{m})_{i,j} K \sqrt{k_{i,j}^2 + k_{y,j}^2} \sin(k_{x,i} x) \cos(k_{y,j} y) \cdot \exp \left\{ \frac{c + \sqrt{c^2 + 4(k_{i,j}^2 + k_{y,j}^2)}}{2} \right\} \cdot \exp \left\{ -\frac{c + \sqrt{c^2 + 4(k_{i,j}^2 + k_{y,j}^2)}}{2} \right\} \right\} \cdot \frac{\sin(k_{y,j} x) \cos(k_{y,j} y)}{z}.$$
where: $K_1$ is the hydraulic conductivity in the Quaternary deposits, $K_2 = K_{0.2} e^{c_2}$ is the hydraulic conductivity in the bedrock, $A_1 = f(c, K_1, K_2, c_{QD}, k_x, k_y)$, $A_2 = f(A_1, c_{QD}, k_x, k_y)$, $c_{QD}$ is the thickness of the Quaternary deposits.

The constant soil depth is a shortcoming because the thickness of Quaternary deposits often differs widely. In areas covered by thick layers of Quaternary deposits, the undulation of the topography is often smaller compared to areas with more shallow deposits. The thickness of the deposits is often determined by the topography, where more material is deposited in lower areas. However, the thickness of the glacial till does not fluctuate much and is seldom larger than four meters. Larger deposits are concentrated to lakes and rivers and consist of well sorted sediments.

3. Result

3.1 Impact of depth dependent hydraulic conductivity

The importance of different scales of topography on the groundwater flow is depth dependence (Fig. 2). The impact of the shorter wavelengths on the groundwater flow decreases faster with depth in relation to longer wavelengths.

![Fig. 2](image)

Fig. 2. The vertical flux at different depths driven by topography of different wavelengths, where diamonds indicates flux at the surface, squares at 500 m depth and triangles at 1000 m depth.

The relationship between topographic scale and water fluxes at various depths change when we account for depth dependent hydraulic conductivity (Fig. 3). Figure 3 also shows that groundwater fluxes generated by topography of larger wavelengths are more affected by the decreasing conductivity than fluxes associated with shorter wavelengths.
Because larger wavelengths dominate the groundwater flow at greater depths, the decrease of conductivity has a relatively larger impact on the absolute flux with depth in bedrock (Table 2).

Table 2

Vertical fluxes (W-flux) at different depths and with different representation of the bedrock hydraulic conductivity

<table>
<thead>
<tr>
<th>Area</th>
<th>Hydraulic conductivity</th>
<th>W-flux (mm/year) at z = 0</th>
<th>W-flux (mm/year) at z = -500</th>
<th>W-flux (mm/year) at z = -1000</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>Constant</td>
<td>121</td>
<td>90</td>
<td>71</td>
</tr>
<tr>
<td></td>
<td>Depth dependent</td>
<td>14</td>
<td>0.51</td>
<td>0.019</td>
</tr>
<tr>
<td>M</td>
<td>Constant</td>
<td>31</td>
<td>13</td>
<td>8.6</td>
</tr>
<tr>
<td></td>
<td>Depth dependent</td>
<td>12</td>
<td>0.18</td>
<td>0.046</td>
</tr>
<tr>
<td>S</td>
<td>Constant</td>
<td>92</td>
<td>25</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>Depth dependent</td>
<td>52</td>
<td>0.42</td>
<td>0.011</td>
</tr>
</tbody>
</table>

3.2 Groundwater recharge through soil-rock interface

The infiltration at z = 0 is found to depend both on the conductivity and the thickness of the Quaternary deposits. As shown in Table 3, a higher conductivity of the QD only increases the flux at the ground surface but at greater depths the flux is only slightly affected. The same effect is created by larger thickness of the deposits (Table 3). The impact of the QDs is independent of the topographic scales (Fig. 4).
Table 3
Sizes and resolution of the study-areas

<table>
<thead>
<tr>
<th>Area</th>
<th>Hydraulic conductivity in QD (m/s)</th>
<th>Depth of QD (m)</th>
<th>U-flux (mm/year) at z = 0</th>
<th>U-flux (mm/year) at z = −50</th>
<th>U-flux (mm/year) at z = −100</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>10⁻⁵</td>
<td>2</td>
<td>30</td>
<td>7.6</td>
<td>4.8</td>
</tr>
<tr>
<td></td>
<td>10⁻⁶</td>
<td>10</td>
<td>104</td>
<td>7.7</td>
<td>4.9</td>
</tr>
<tr>
<td>M</td>
<td>10⁻⁵</td>
<td>2</td>
<td>30</td>
<td>7.6</td>
<td>4.8</td>
</tr>
<tr>
<td></td>
<td>10⁻⁶</td>
<td>10</td>
<td>104</td>
<td>7.7</td>
<td>4.9</td>
</tr>
<tr>
<td>S</td>
<td>10⁻⁵</td>
<td>2</td>
<td>250</td>
<td>24</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td>10⁻⁹</td>
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<tr>
<td></td>
<td>10⁻⁶</td>
<td>2</td>
<td>72</td>
<td>24</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td>10⁻⁶</td>
<td>10</td>
<td>121</td>
<td>24</td>
<td>13</td>
</tr>
</tbody>
</table>

Fig. 4. The vertical flux at the groundwater surface driven by topography of different wavelengths, of four different representations of the Quaternary deposits: two different depths, 2 and 10 m, combined with two different hydraulic conductivities, 10⁻⁵ and 10⁻⁶.

4. Discussion and conclusions
Landscape topography is the most important driving force for groundwater flow and all scales of topography contribute to groundwater movement. At the groundwater surface the contribution to the groundwater flow is rather equal for all scales (Fig. 2),
but the impact of shorter topographical scales decay faster with depth than longer scales.

The depth dependent hydraulic conductivity is another important factor for the groundwater movement. By controlling how deep the groundwater flow cells become, it also determines the residence time for groundwater. In addition, it controls how different scales of topography affect the groundwater flow at different depths and even at the surface. The decreasing hydraulic conductivity with depth blocks out more effectively the impact of the larger landscape features compared to the smaller features. This is most obvious at flows at great depths \((z = −1000 \text{ m})\), but the effect is also present at the surface.

The recharge of groundwater is driven by topography, but the physical properties of Quaternary deposits are controlling the magnitude of the recharge. The higher conductivity of the Quaternary deposits creates a much larger infiltration rate compared to a geological representation of the subsurface where the bedrock reaches up all the way to the ground surface. However, even if the infiltration is increased up to a hundred times, fluxes at 50 m depth are not significantly affected. We note here that this study was performed with a depth dependent hydraulic conductivity in the bedrock and most likely the effects of the Quaternary deposits would reach deeper with a homogeneous representation of the bedrock but the depth dependency of the hydraulic conductivity is more realistic in most areas (Ingebritsen and Manning 1999).

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References

Zijl, W., 1999, Scale aspects of groundwater flow and transport systems, Hydrogeology J., 7, 139-150.