Modelling of a Two-Dimensional Velocity Field for the Water Flow in the Lake of Dobczyce

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Abstract
This article describes an attempt to model the water flow velocity field for the Dobczyce retention water body. To solve the case, a finite element approach is used. Mathematical model and observational base are described, as well as their application to the problem. Potential difficulties are outlined. First results are shown along with the plans for further research and possibilities of practical applications of the generated data.

1. Introduction
Rapid development of computational techniques and fast increase of computing power available to engineers which happened during the recent couple of years allow us to apply mathematical models to more and more complex objects. We are able not only to simulate the behavior of abstract simplified systems, but nowadays we are capable to compute – with reasonable accuracy – how much more compound “real life” systems behave.

The aim of this article is to show an approach to simulate the water flow velocity field for the whole Dobczyce lake; an attempt is based on actual data gathered during a series of topographic and bathymetric measurements. The results to be obtained are intended to constitute the fundamentals for practical environmental engineering applications including pollution spreading prognoses, sediment and rubble transport predictions, banks erosion warning systems and so on. This paper shows the theoretical and observational basis of the model under development along with some recently obtained first results.

2. The lake of Dobczyce
The Dobczyce lake is a retention reservoir placed at 60th kilometer of the Raba river. Table 1 sums up some basic data about that lake (Nachlik et al. 2006). (ODGW, un-
dated) and Fig. 1 shows the shape of the lake for minimal, nominal and maximal water levels.

Table 1
Dobczyce Lake – basic data

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total capacity</td>
<td>$14.5 \div 125 \times 10^6$ m$^3$</td>
</tr>
<tr>
<td>Flooded area</td>
<td>$3.35 \div 10.65$ km$^2$</td>
</tr>
<tr>
<td>Total flow (possible range)</td>
<td>$1.8 \div 2717$ m$^3$/s</td>
</tr>
<tr>
<td>Yearly average flow</td>
<td>$10.6$ m$^3$/s</td>
</tr>
<tr>
<td>Design (0.3%) flow</td>
<td>$1560$ m$^3$/s</td>
</tr>
<tr>
<td>Range of surface level changes</td>
<td>$15.9$ m</td>
</tr>
<tr>
<td>Average depth (at average water level)</td>
<td>$10.2$ m</td>
</tr>
<tr>
<td>Covered watershed area</td>
<td>$768$ km$^2$</td>
</tr>
</tbody>
</table>

Fig. 1. The shape of the Dobczyce lake at different water levels: minimal (light grey), average (grey), and maximal (dark grey). The symbol 1 shows the position of dam inlets while 2 denotes the location of the water supply inlet.

The main inflow into the lake is the Raba river. There are several streams that also flow into this lake (out of which Wolnica is the most important one), but their contribution to the total flow is less than 5% and most of them can be neglected during the preliminary analysis. There is a number of outflows from the lake:
- Four bottom sluices at the base of the dam;
- A three-section open spillway in the dam;
- One power plant sluice close to the dam;
- One water supply sluice placed about 0.5 km from the dam.

As the considered water body occupies a flooded mountain valley, its banks are well developed: their slope varies greatly and their shape is complex (as shown on Fig. 1); the total length of the banks (for average water level) is about 40 km. The bed bathymetry is also multifaceted. For all the calculations described in this article a digital GPS-based bathymetric map of the area (Mazoń et al. 1998) has been used.

### 3. Governing equations and the solving method

The equations used to model the flow of water in the considered lake are based on mass and momentum conservation concepts. They are reduced to two-dimensional ones as the vertical movement can be neglected (Froehlich 2003). Thus, the vertical \( z \) dimension is treated as a parameter the \( x, y \)-plane velocity is dependent on. The velocity components for both horizontal coordinates are then:

\[
V_x = \frac{1}{H} \int_{z_0}^{z} v_x \, dz; \quad V_y = \frac{1}{H} \int_{z_0}^{z} v_y \, dz
\]

where: \( V_x, V_y = \) averaged (2D) velocity components in appropriate directions, \( H = \) water depth, \( z_0 = \) bed elevation, \( z = \) surface elevation, \( v_x, v_y = \) real (3D) horizontal velocity components in appropriate directions.

After the integration, the continuity (mass conservation) equation takes the following form:

\[
\frac{\partial z}{\partial t} + \frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} = q_m
\]

where: \( q_x = V_x H = \) unit flow rate in the \( x \) direction, \( q_y = V_y H = \) unit flow rate in the \( y \) direction, \( q_m = \) mass flow rate per unit area (positive for inflow, negative for outflow).

Momentum transport equations for both horizontal directions are symmetrical to each other. That in the \( x \)-direction is as follows:

\[
\frac{\partial q_x}{\partial t} + \frac{gH \partial z}{\partial x} + \frac{H \partial p_a}{\partial x} + \frac{\partial}{\partial x} \left( \frac{\beta q_x}{H} + \frac{gH^2}{2} \right) + \frac{\partial}{\partial y} \left( \frac{\beta q_y}{H} \right)
\]

\[\frac{1}{\rho} \left( \tau_{hx} - \tau_{sx} - \frac{\partial}{\partial x} H \tau_{sx} - \frac{\partial}{\partial y} H \tau_{sy} \right) - \Omega q_y = 0\]

where: \( g = \) Earth gravity, \( \rho = \) water density (considered constant), \( p_a = \) atmospheric pressure at the surface level, \( \beta = \) momentum correction coefficient, \( \tau_{hx} = \) bed shear stress (\( x \) component), \( \tau_{sx} = \) surface shear stress (\( x \) component), \( \tau_{sy}, \tau_{xy} = \) turbulence
shear stresses acting in the x direction on planes perpendicular to the x and y directions respectively, $\Omega$ = Coriolis parameter responsible for Earth rotation effects.

In order to simplify the model, the Coriolis effect, the atmospheric pressure variability, and the surface stress (usually caused by wind) are considered very small and their appropriate formulas are taken out of the equations to solve. Moreover, the momentum flux is taken without any corrections ($\beta = 1$). The simplified equation is then:

$$\frac{\partial q_s}{\partial t} + gH \frac{\partial z_0}{\partial x} + \frac{\partial}{\partial x} \left( \frac{q_s}{H} + \frac{gH^2}{2} \right) + \frac{\partial}{\partial y} \left( q_s q_y \right)$$

$$+ \frac{1}{\rho} \left( \tau_{ss} - \frac{\partial H \tau_{sx}}{\partial x} - \frac{\partial H \tau_{sy}}{\partial y} \right) = 0$$

(4)

The bed stress is calculated as follows:

$$\tau_{bx} = \frac{pcmq_s \sqrt{(q_s^2 + q_y^2)}}{H^2}; \quad c = \frac{gn^2}{\sqrt{H}}; \quad m = \sqrt{1 + \left( \frac{\partial z_0}{\partial x} \right)^2 + \left( \frac{\partial z_0}{\partial y} \right)^2}$$

(5)

where $n$ = Manning roughness coefficient.

For turbulent stresses, the following general formula is used:

$$\tau_{xy} = \rho V_{i} \left( \frac{\partial V_x}{\partial \varphi} + \frac{\partial V_y}{\partial \psi} \right)$$

(6)

where $\varphi$ and $\psi$ represent any coordinate symbol, and:

$$v_i = \left[ \frac{m}{s^2} \right] + 0.1 \sqrt{\left( \frac{\partial V_x}{\partial x} \right)^2 + \left( \frac{\partial V_y}{\partial y} \right)^2 + \left( \frac{\partial V_x}{\partial y} + \frac{\partial V_y}{\partial x} \right)^2}.$$  

(7)

The whole method is called “two-dimensional depth-averaged flow analysis” or sometimes “two-and-a-half-dimensional flow analysis”; see e.g. (Zienkiewicz and Taylor 2002, A, p. 219-223, 237-239). In order to solve its equations the finite element method is used. The calculation performed on a discrete mesh consisted of quadrilaterals (wherever possible) and triangles. The procedure is supplemented by the method of weighted residuals – see e.g. (Zienkiewicz and Taylor 2002, B, p. 42-60) in order to provide better convergence.

To execute the calculations the “Depth-averaged Flow and Sediment Transport Model – FESWMS” program (Froehlich 2003) has been chosen due to its high versatility and stability. This tool is also capable of calculating sediment transport and pollution spreading which may be crucial in further applications of the results. (It is a part of the Surface Water Modeling System “SMS” by the EMS-i company, obtainable at [www.ems-i.com](http://www.ems-i.com)).
4. Finite element mesh construction

Due to the parameters of considered water body (its complex bathymetry, wide range of water level and flow rate values) it is virtually impossible to construct one finite element mesh that covers all the possible states of the lake. Even using the “element drying/wetting” option embedded in the program does not solve the case for high flow and bed slope values. The aim is then to construct a set of meshes: each of them working properly for the whole flow range in a small bracket (one meter or so) of water level value.

There are still several difficulties one must face when constructing such a mesh for the Dobczyce lake. First of all the water body could not be considered uniform; it consists of several zones that should be treated differently. These parts are: Raba inflow zone, dam zone, long northern valley, shallow southeastern basin, bank zones, and finally the main deep body of the lake. While the last of the listed zones is relatively easy to cover with a mesh of fairly big elements (as too many small elements there make the calculations much longer and tend to cause the solutions to diverge for certain flow values), the first six zones require careful creation of meshes having smaller elements fitting crucial bathymetric features of the given region (like sluices positions or areas of rapid depth changes).

This leads to another construction problem: in the non-central regions the intelligent mesh generation and optimization tools usually fail – leading to unstable or even diverging models. Significant fraction of elements has to be manually designed and semi-automatically created. All these partial meshes have then to fit each other not to cause computational troubles at the junction nodes.

Finally the mesh needs to be checked whether it provides stable and consistent (changing continuously) results in the whole range of boundary condition values (mostly inflow/outflow rates). The obtained results also should not change rapidly for small changes of the model parameters (like eddy viscosity or Manning roughness coefficients). Moreover, it should be checked if two meshes created for neighboring water level values (e.g. 1m difference) give similar flow maps as their output for similar boundary conditions imposed.

5. First results

Applying the described methodology to the Dobczyce lake case has brought us promising outcome so far. This chapter presents some of the early obtained results. The mesh has been made to work for high water levels (about 272 meters above sea level), and the velocity field simulation boundary conditions have been set to a 0.3% flood (reliable flow, 1560 m$^3$/s total). All the outflows are considered working (with their appropriate effectiveness) and 8 streams (besides the Raba river) are considered as contributing to the total inflow. The mesh used is shown in Fig. 2.

Figure 3 shows a map of the resulting velocity field. Shading of any given area is proportional to the logarithm of the water velocity there. Obtained picture passes a common sense test as well as a comparison to the results of simple qualitative kinematical analysis. A series of observational comparisons is planned when velocity maps for various flow values and for several water levels are ready.
Fig. 2. The mesh used for calculating flows for the water level of about 272 meters above the sea limit. It consists of 2187 quadrilateral elements and 2274 triangular ones. The number of nodes is 14389.

Fig. 3. Velocity map for 1560 m³/s flow at maximal water level. Arbitrary units, logarithmic scale on the $z$ axis.

A more detailed view of the lower part of the lake is presented on Fig. 4. Along with the shading (still proportional to the velocity logarithm) a grid of arrows is introduced showing the flow direction in the points where the arrows are placed. It can be easily seen that whirls appear alongside the areas where the main current turns sideways. Such behavior is – again – intuitively expected. It should not be also difficult to check whether such phenomena occur in the actual lake.
6. Conclusions and perspectives

The finite element method described in Chapter 3 and embedded in the FESWMS program proved to be useful in calculating water flow in the lake of Dobczyce at least as long as computational meshes are constructed with care. One such mesh can be used to perform calculations for the whole possible range of flow values but only in very limited bracket of water level. First obtained results are reasonable and consistent; they well qualify for further analysis and calibration. They show characteristic features of the velocity field that should subject to observational verification leading to consecutive tuning of the model.

Among various possible uses of generated velocity maps two should be outlined now: First – prognoses obtained for high flow values could give us a clue about such phenomena as rubble transport and banks/bed erosion during floods. Second – results computed for average flows could lead us to better understanding of sediment and pollution transport in the considered retention water body working under normal, average circumstances. This is very important because of the presence of water supply system inlet and several recreation zones in this area, that have to be appropriately protected against any contamination. Water flow maps may help in determining locations and sizes of necessary protective zones.

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