REFLECTION/TRANSMISSION OF PLANE SH WAVE THROUGH A SELF-REINFORCED ELASTIC LAYER BETWEEN TWO HALF-SPACES

Sushil CHAUDHARY¹, Ved P. KAUSHIK¹ and Sushil K. TOMAR²

¹Department of Mathematics, Kurukshetra University, Kurukshetra -136119, India
²Department of Mathematics, Panjab University, Chandigarh -160014, India
e-mail: sktomar@yahoo.com

Abstract

The reflection and transmission of plane SH wave through a self-reinforced elastic layer sandwiched between two homogeneous visco-elastic solid half-spaces has been studied. The expressions for reflection and transmission coefficients are obtained. These coefficients are computed against the angle of incidence, normalized wave number and the reinforced parameters. The results obtained have been presented graphically.

The problem studied by Borcherdt has been reduced as a special case of our problem. It is found that reflection and transmission coefficients depend on the angle of incidence, normalized wave number and the reinforcement parameters of the sandwiched layer.

Key words: SH wave, visco-elastic half-space, self-reinforced elastic layer, reflection coefficient, transmission coefficient.

1. INTRODUCTION

The knowledge of elastic wave propagation through the earth is helpful in investigating the internal structure of the earth and exploration of valuable minerals buried. The seismic signals propagating through the earth medium have to travel through various types of minerals present in the earth in the form of layers. Doubtless to say that the
elastic properties of these layered materials do affect the propagation of the seismic waves. Moreover, the discontinuities present in the earth between the layers produce the reflection and refraction phenomena.

The subject of propagation, reflection and refraction of the elastic waves from the boundary surfaces is very important and researchers have investigated several problems by assuming different models. The propagation and reflection of general plane $SH$ wave in homogeneous visco-elastic media has been studied, e.g., by Cooper (1967), Schoenberg (1971), Borcherdt (1977; 1982). Buchan (1971a; b) studied the propagation of plane $SH$ waves in a linear visco-elastic solid. The problem of transmission and reflection of inhomogeneous plane $SH$ waves at an interface between homogeneous and inhomogeneous visco-elastic half-spaces has been studied by Kaushik and Chopra (1980). Kaushik and Chopra (1983) studied the reflection and transmission of general $SH$ waves through $n$-layered visco-elastic media. Biswas (1967) studied the transmission of $SH$ waves through a visco-elastic layer embedded between two elastic half-spaces and concluded that the effect of imaginary components of elastic properties on the transmission of waves increases as the frequency increases. Shaw and Bugl (1968) studied the transmission of time harmonic plane $P$ and $SV$ waves in a layered infinite linear visco-elastic medium. The generation and propagation of $SH$-type waves due to stress discontinuity in a linear visco-elastic layered medium was studied by Pal and Kumar (1995). Pipkin and Rogers (1971) developed the plane strain theory of finite deformation for fiber-reinforced materials. Belfield et al. (1983) gave a linear model for such a material, which was later used by Verma and Rana (1983) for studying the problem of rotation of a circular cylindrical tube reinforced by fibers lying along helices. Verma (1986) examined the influence of an external magnetic field on the propagation of purely transverse waves polarized parallel to the plane faces in a homogeneous, initially unstressed, infinitely conducting, self-reinforced elastic flat plate extending to infinity. Verma et al. (1988) studied the problem of magneto-elastic transverse surface waves in self-reinforced elastic solids. Green (1991) studied the problem of reflection and transmission phenomenon of transient stress waves in fiber composite laminates. Chattopadhyay and Chaudhury (1995) studied the propagation of magneto-elastic shear waves in an infinite self-reinforced plate. The problem of surface waves in fiber-reinforced anisotropic elastic media was studied by Sengupta and Nath (2001).

It is well known in the literature that the earth medium exhibits various types of elastic properties. The hard and/or soft rocks, which may be self-reinforced in nature, might be present beneath the earth surface. These rocks when come in the way of seismic waves do affect their propagation and such seismic signals are always influenced by the elastic properties of the media through which they travel. Keeping in view this type of geophysical situation, we analyze the problem of reflection and refraction of plane $SH$ waves using the model of a self-reinforced elastic layer sandwiched between two homogeneous visco-elastic half-spaces.
2. FORMULATION OF THE PROBLEM AND BASIC EQUATIONS

Consider a layer $M_{II}$ having width $d$ of a perfectly conducting self-reinforced linearly elastic solid between two homogeneous visco-elastic half-spaces $M_I$ and $M_{III}$. Let $Z = 0$ be the interface between $M_I$ and $M_{II}$ and $Z = -d$ between $M_{II}$ and $M_{III}$. The $Z$-axis of the rectangular coordinate system is pointing into the lower half-space $M_I$. A plane $SH$ wave is assumed to be incident at an angle $\theta_1$. The geometry of the problem is shown in Fig. 1. We shall denote the density and complex rigidity, respectively, by $\rho_1$, $\mu_1$ in $M_I$ and by $\rho_3$, $\mu_3$ in $M_{III}$. We postulate that an incident $SH$ wave will give rise to: (i) a reflected $SH$ wave at an angle $\theta_1$ in $M_I$, (ii) a refracted $SH$ wave at an angle $\theta_2$ in the layer $M_{II}$, and (iii) the refracted $SH$ wave in $M_{II}$ which will again reflect the same wave from the interface $Z = -d$ along with a transmitted $SH$ wave at an angle $\theta_3$ in the half-space $M_{III}$.

The equations of motion in a self-reinforced perfectly conducting medium $M_{II}$ having only the electromagnetic force, that is, Lorentz force $J \times B$, are given by

$$
\tau_{ij,j} + (J \times B)_i = \rho \frac{\partial^2 u_i}{\partial t^2}, \quad (i, j = 1, 2, 3), \quad (1)
$$

where $\tau_{ij}$ are the Cartesian components of the stress tensor, $J$ is the electric current, $B$ is the magnetic induction, $u_i$ are the components of displacement vector $u$, $\rho$ is the mass density and comma denotes the partial derivative with respect to space coordinates.

$J$ and $B$ are given by the well-known Maxwell’s fundamental equations:

$$
\nabla \times H = J, \quad \nabla \times E = -\frac{\partial B}{\partial t}, \quad \nabla \cdot B = 0, \quad B = \mu H, \quad (2)
$$

$$
J = \sigma \left[ E + \frac{\partial u}{\partial t} \times B \right]. \quad (3)
$$
Also, the Maxwell’s stress tensor is given by

\[
\tau_{ij}^\mu = \mu_e(H_j h_i + H_i h_j - H_i \delta_{ij}) \quad ,
\]

where \( E \) is the induced electric field, \( H = (H_x, H_y, H_z) \) is the total applied and induced magnetic field, \( \mu_e \) is the magnetic permeability, \( \sigma \) is the electrical conductivity, and \( \delta \) is the change in the basic magnetic field. Following Belfield et al. (1983), the stress-strain relation in medium \( M_{II} \), in which reinforcement is to make the material locally transversely isotropic, whose preferred direction is that of unit vector \( a \), is given by

\[
\tau_{ij} = \lambda e_{ij} \delta_{ij} + 2\mu_e e_{ij} + \alpha(a_a a_a e_m \delta_{ij} + e_{ij} a_a)
\]

\[
+ 2(\mu_e - \mu_L)(a_a a_a e_m + a_e a_e) + \beta a_a a_a e_m a_a \delta_{ij} \quad ,
\]

where \( \lambda, \mu_e, \mu_L, \alpha \) and \( \beta \) are the material constants with the dimension of stress, and other symbols have their usual meanings. Using eqs. (2) and (3) we can write

\[
\nabla^2 H = \sigma \mu_e \left[ \frac{\partial H}{\partial t} - \nabla \times \left( \frac{\partial \mathbf{u}}{\partial t} \times \mathbf{H} \right) \right] .
\]

For the motion of \( SH \) wave, we take \( \mathbf{u} = (0, V_2, 0) \) and since we are considering a two-dimensional problem in \( X-Z \) plane, we have \( \partial / \partial Y \equiv 0 \). Thus, from eq. (6) we get

\[
\frac{\partial H_x}{\partial t} = \frac{1}{\sigma \mu_e} \nabla^2 H_x \quad ,
\]

\[
\frac{\partial H_y}{\partial t} = \frac{1}{\sigma \mu_e} \nabla^2 H_y + \frac{\partial}{\partial X} \left( H_x \frac{\partial V_2}{\partial t} \right) + \frac{\partial}{\partial Z} \left( H_z \frac{\partial V_2}{\partial t} \right) \quad ,
\]

\[
\frac{\partial H_z}{\partial t} = \frac{1}{\sigma \mu_e} \nabla^2 H_z .
\]

For perfect conductor, i.e., when \( \sigma \to \infty \), we have from eqs. (7a, b, c)

\[
\frac{\partial H_x}{\partial t} = \frac{\partial H_y}{\partial t} = 0 \quad ,
\]

\[
\frac{\partial H_y}{\partial t} = \frac{\partial}{\partial X} \left( H_x \frac{\partial V_2}{\partial t} \right) + \frac{\partial}{\partial Z} \left( H_z \frac{\partial V_2}{\partial t} \right) .
\]

It is clear from eq. (8a) that there is no perturbation in \( H_x \) and \( H_z \). Therefore, we may take small perturbation \( h_2 \) in \( H_y \), so that

\[
H_x = H_{\alpha_1} \quad , \quad H_y = H_{\alpha_2} + h_2 \quad , \quad H_z = H_{\alpha_3} .
\]

Taking

\[
H_{\alpha_1} = H_\alpha \cos \phi \quad , \quad H_{\alpha_2} = 0 \quad , \quad H_{\alpha_3} = H_\alpha \sin \phi .
\]
where \( H_0 = |H_0| \) and \( \phi \) is the angle at which the wave crosses the magnetic field. Hence,

\[
H = (H_0 \cos \phi, h_2, H_0 \sin \phi).
\]  

(10)

Using eq. (10) into eq. (8b), and integrating with respect to \( t \) we get

\[
h_2 = H_0 \left( \frac{\partial V_x}{\partial X} \cos \phi + \frac{\partial V_z}{\partial Z} \sin \phi \right).
\]  

(11)

Since the initial value of \( h_2 \) is zero, we get from eq. (2)

\[
J \times B = \mu \left[ \left( \mathbf{H} \cdot \nabla \right) \mathbf{H} - \frac{1}{2} \nabla H^2 \right].
\]  

(12)

Using eq. (10) into eq. (12), we get

\[
(J \times B)_1 = (J \times B)_3 = 0,
\]  

(13a)

\[
(J \times B)_2 = \mu H_0 \left( \frac{\partial h_0}{\partial X} \cos \phi + \frac{\partial h_0}{\partial Z} \sin \phi \right).
\]  

(13b)

Equation (13b) with the aid of eq. (11) yields

\[
(J \times B)_2 = \mu H_0 \left( \frac{\partial V_x}{\partial X} \cos \phi + \frac{\partial V_z}{\partial Z} \sin \phi \right).
\]  

(14)

Using \( a = (a_1, 0, a_3) \) into eq. (5), the non-zero stress components are obtained as

\[
\tau_{23} = \mu_T \frac{\partial V_x}{\partial Z} + \left( \mu_L - \mu_T \right) \left( a_1 \frac{\partial V_z}{\partial Z} + a_3 \frac{\partial V_z}{\partial X} \right),
\]  

(15a)

\[
\tau_{31} = \mu_T \frac{\partial V_x}{\partial X} + \left( \mu_L - \mu_T \right) \left( a_1 \frac{\partial V_z}{\partial X} + a_3 \frac{\partial V_z}{\partial Z} \right).
\]  

(15b)

Using expressions given by eqs. (14) to (15b) into (1), we obtain

\[
\left[ \mu_T + \mu_L H_0^2 \sin^2 \phi + a_3^2 \left( \mu_L - \mu_T \right) \right] \frac{\partial^2 V_z}{\partial Z^2} \]

\[
+ \left[ 2a_1 a_3 \left( \mu_L - \mu_T \right) + \mu_L H_0^2 \sin 2\phi \right] \frac{\partial^2 V_z}{\partial X \partial Z} \]

\[
+ \left[ \mu_T + \mu_L H_0^2 \cos^2 \phi + a_1^2 \left( \mu_L - \mu_T \right) \right] \frac{\partial^2 V_z}{\partial X^2} = \rho \frac{\partial^2 V_z}{\partial t^2}.
\]  

(16)

For time harmonic wave propagating in positive \( X \)-direction, the solution of eq. (16) may be taken as
where $A_2$ and $B_2$ are unknown and

$$\gamma_2 = \sqrt{\frac{S^2}{4P^2} + \frac{1}{P} \frac{\rho \omega^2}{k_1^2} - Q}, \quad S = 2a_i a_3 (\mu_e - \mu_r) + \mu_e H_0^2 \sin 2\phi,$$  \hfill (18a)

where $P = \mu_r + \mu_e H_0^2 \sin^2 \phi + a_i^2 (\mu_e - \mu_r), \quad Q = \mu_r + \mu_e H_0^2 \cos^2 \phi + a_i^2 (\mu_e - \mu_r).$ \hfill (18b)

Using eq. (17) into eq. (15a), the stress components are

$$\tau_{23} = \left[ P \left\{ i k \left( \gamma_2 - \frac{S}{2P} \right) A_2 \exp \left[ i k \left( \gamma_2 - \frac{S}{2P} \right) Z \right] - i k S \left\{ A_2 \exp \left[ i k \left( \gamma_2 - \frac{S}{2P} \right) Z \right] + B_2 \exp \left[ -i k \left( \gamma_2 + \frac{S}{2P} \right) Z \right] \right\} \exp \left[ i(\omega t - kX) \right] - (\tau_{23})^\mu \right], \quad (19)$$

where $(\tau_{23})^\mu = \mu_e H_0^2 \sin \phi \left( \frac{\partial V_z}{\partial X} \cos \phi + \frac{\partial V_z}{\partial Z} \sin \phi \right).$

Following Kaushik and Chopra (1982), the equation of motion governing $SH$-type of motion in $M_i$ ($i = I, III$) is given by

$$V^2 V_i + k_{ij}^2 V_i = 0,$$ \hfill (20)

where $k_{ij}^2 = \rho_i \omega^2 / \mu_i$ and the complex displacement vector in medium $M_i$ and $M_{III}$ is given by

$$V_i = \left( 0, V_y, 0 \right) \exp (i \omega t).$$ \hfill (21)

The general solution of eq. (20) is of the form

$$V_i = \sum_{j=1}^{2} D_j \exp (-A_{ij} \cdot r) \exp \left[ i(\omega t - P_{ij} \cdot r) \right] \hat{y},$$ \hfill (22)

where $D_j$ are the complex amplitudes, $r$ is the position vector and $\hat{y}$ is a unit vector in the $Y$-direction. The propagation and attenuation vectors $P_{ij}$ and $A_{ij}$ are defined, respectively, by

$$P_{ij} = k_{Ri} \hat{x} + (-1)^i d_{bij} \hat{z}, \quad A_{ij} = -k_{Ri} \hat{x} + (-1)^{i+1} d_{bij} \hat{z},$$ \hfill (23)

where $k_i = k_R + ik_1$ is the complex wave number, $\hat{z}$ is a unit vector in the $Z$-direction, subscripts $R$ and $I$ denote the real and imaginary parts and $d_{bij}^2 = k_{bij}^2 - k_i^2.$ The vectors $P_{ij}$ and $A_{ij}$ satisfy
\[ \mathbf{P}_y \cdot \mathbf{P}_y - \mathbf{A}_y \cdot \mathbf{A}_y = R_y k_{\beta}^2, \quad \mathbf{P}_y \cdot \mathbf{A}_y = |\mathbf{P}_y| \cdot |\mathbf{A}_y| \cos \gamma_y = -\frac{1}{2} I_y k_{\beta}^2. \]  

From eqs. (24), we can write

\[ |\mathbf{P}_y| = \left\{ \frac{1}{2} \left[ R_y k_{\beta}^2 + \left( R_y k_{\beta}^2 \right)^2 + \frac{\left( I_y k_{\beta}^2 \right)^2}{\cos^2 \gamma_y} \right] \right\}^{1/2}, \tag{25} \]

\[ |\mathbf{A}_y| = \left\{ -\frac{1}{2} \left[ R_y k_{\beta}^2 + \left( R_y k_{\beta}^2 \right)^2 + \frac{\left( I_y k_{\beta}^2 \right)^2}{\cos^2 \gamma_y} \right] \right\}^{1/2}, \tag{26} \]

where \( \gamma_y \) is the angle between the propagation vector \( \mathbf{P}_y \) and attenuation vector \( \mathbf{A}_y \).

### 3. SPECIFICATION OF THE PROBLEM

Since the SH wave is made incident at the interface \( Z = 0 \) obliquely in the medium \( M_1 \), the displacement in \( M_1 \) is given by

\[ \mathbf{V}_i = \begin{bmatrix} D_{1i} \exp(-\mathbf{A}_{1i} \cdot \mathbf{r}) \exp(i(\omega t - \mathbf{P}_{1i} \cdot \mathbf{r})) \\
+ D_{12} \exp(-\mathbf{A}_{12} \cdot \mathbf{r}) \exp(i(\omega t - \mathbf{P}_{12} \cdot \mathbf{r})) \end{bmatrix} \mathbf{y}, \tag{27} \]

where \( D_{1i} \) is the amplitude of incident SH wave and \( D_{12} \) is that of reflected SH wave.

In medium \( M_{II} \), the displacement and stresses are given by eqs. (17) and (18b). In the half-space \( M_{III} \), the displacement is given by

\[ \mathbf{V}_i = D_{31} \exp(-\mathbf{A}_{31} \cdot \mathbf{r}) \exp(i(\omega t - \mathbf{P}_{31} \cdot \mathbf{r})) \mathbf{y}, \tag{28} \]

where \( D_{31} \) is the complex amplitude of the transmitted wave.

The reflection and transmission coefficients of SH wave will be determined by the boundary conditions to be satisfied at the interfaces. These conditions are the continuity of displacement and stresses at the interfaces \( Z = -d \) and \( Z = 0 \). Mathematically, these boundary conditions are

\[ V_3 = V_2, \quad \left[ \mu \frac{\partial V_3}{\partial Z} \right]_{III} = \left[ \tau_{23} \right]_{II} \quad \text{at} \quad Z = -d, \tag{29a} \]

\[ V'_3 = V'_2, \quad \left[ \mu \frac{\partial V'_3}{\partial Z} \right]_{I} = \left[ \tau_{23} \right]_{II} \quad \text{at} \quad Z = 0. \tag{29b} \]

Also, Snell’s law is given by

\[ c = \frac{\beta_1}{\sin \theta_1} = \frac{\beta_2}{\sin \theta_2} = \frac{\beta_3}{\sin \theta_3}, \tag{30} \]
where $\beta_i^2 = \mu_i / \rho$ and $\beta_1, \beta_3$ are the complex shear wave velocities in the visco-elastic half-spaces. Following Borcherdt (1977), the quantities $k_1$ and $k_2$ obey the following relation:

$$k = |P_{11}| \sin \theta_1 - i |A_{11}| \sin(\theta_1 - \gamma_1) = k_1 = k_2,$$

(31)

where $P_{11}$ and $A_{11}$ are the propagation and the attenuation vectors in $M_i$, and $\theta_1$ and $\gamma_1$ are the angle of incidence and the angle between $P_{11}$ and $A_{11}$, respectively.

Using eqs. (17), (18b), (27) and (28) into the above boundary conditions given by (29) and Snell’s law (30), we shall obtain the four simultaneous equations. From these equations, one can obtain the following formulae for reflection and transmission coefficients:

$$R = A_i / A , \quad T = A_x / A ,$$

(32)

where $R = D_{32} / D_{11}$, $T = D_{31} / D_{11}$.

$$\Delta = \left\{ \left( d_1 - \frac{\mu_1 d_\rho_1}{k} \right) \left( 1 - \frac{k d_2}{k} \right) \exp \left[ -ik \left( \gamma_2 - \frac{S}{2P} \right) d \right] + \left( d_2 + \frac{\mu_3 d_\rho_3}{k} \right) \left( \frac{k d_1}{\mu_1 d_\rho_1} + 1 \right) \exp \left[ ik \left( \gamma_2 + \frac{S}{2P} \right) d \right] \exp \left[ -id_\rho_1 d \right] \right\},$$

(33a)

$$\Delta_i = \left\{ \left( d_1 - \frac{\mu_1 d_\rho_1}{k} \right) \left( 1 + \frac{k d_2}{k} \right) \exp \left[ -ik \left( \gamma_2 - \frac{S}{2P} \right) d \right] + \left( d_2 + \frac{\mu_3 d_\rho_3}{k} \right) \left( \frac{k d_1}{\mu_1 d_\rho_1} - 1 \right) \exp \left[ ik \left( \gamma_2 + \frac{S}{2P} \right) d \right] \exp \left[ -id_\rho_1 d \right] \right\},$$

(33b)

$$\Delta_x = 2 (d_1 + d_2) \exp \left( ik \frac{S}{P} d \right).$$

(33c)

4. SPECIAL CASE

When the thickness of the layer is zero, the problem reduces to the problem of reflection and refraction of $SH$ wave at a plane interface between two homogeneous visco-elastic half-spaces in welded contact. Thus, putting $d = 0$ into eqs. (32), the following expressions for reflection and transmission coefficients are obtained:

$$R = \frac{\mu_1 d_\rho_1 - \mu_3 d_\rho_3}{\mu_1 d_\rho_1 + \mu_3 d_\rho_3} , \quad T = \frac{2\mu_1 d_\rho_1}{\mu_1 d_\rho_1 + \mu_3 d_\rho_3} .$$

(34)
These are the same formulae for the reflection and transmission coefficients as obtained earlier by Borcherdt (1977) for the relevant problem.

5. NUMERICAL CALCULATION

For the purpose of numerical calculations, we introduce the following non-dimensional parameters:

\[ Q_1^{-1} = \frac{\mu_{1i}}{\mu_{1R}}, \quad Q_3^{-1} = \frac{\mu_{3i}}{\mu_{3R}}, \quad \xi = \frac{\mu_{1R}}{\mu_{3R}}, \]

where \( \mu_1 = \mu_{1R} + \mu_{1I} \), \( \mu_3 = \mu_{3R} + \mu_{3I} \), so that \( \beta_3^i = \mu_3 / \rho_3 \), \( (i = 1, 3) \), and \( Q_1^{-1} \) and \( Q_3^{-1} \) are the loss parameters in the medium \( \text{M}_1 \) and \( \text{M}_{\text{III}} \), respectively.

The numerical computations have been performed using the following values of non-dimensional material parameters: \( Q_1 = 45 \), \( Q_3 = 30 \), \( \mu_1 / \mu_3 = 2 \), \( \rho_3 / \rho_1 = 0.7 \), \( \rho_2 / \rho_1 = 0.9 \), \( \mu_3 / \mu_2 = 1.6 \), \( \mu_3 / \mu_1 = 0.5 \), \( c / \beta_3^R = 0.5 \), \( c / \beta_3^I = 0.8 \). With these values we have computed the absolute values of reflection and transmission coefficients.

![Graph](image.png)

**Fig. 2.** Variation of reflection coefficient \( R \) versus angle of incidence \( \theta \). Curves 1–4 when \( kd = 0.1, 0.2, 0.3, 0.4 \), respectively; \( \phi = 30^\circ \), \( \epsilon = 0.5 \), \( a_1 = 0.4 \), \( a_3 = 0.925 \).

Figure 2 shows the variation of reflection coefficient with angle of incidence \( \theta_i \) (hereafter \( \theta \)) for different values of \( kd \). We notice that as \( kd \) takes the values 0.1, 0.2, 0.3, 0.4, the reflection coefficient increases for all values of angle of incidence, except
for few values of \( \theta \) near the normal incidence. Near the normal incidence, the reflection coefficient fluctuates and the magnitude of fluctuation increases with increasing value of \( kd \). Also, in the neighbourhood of \( \theta = 70^\circ \), the reflection coefficient is almost independent of \( kd \).

Figure 3 shows the variation of transmission coefficient with angle of incidence for different values of \( kd \). Note that at \( \theta = 1^\circ, 72^\circ \) and \( 90^\circ \), the transmission coefficient does not depend on \( kd \). However, at \( \theta = 1^\circ \) and \( 90^\circ \), the transmission coefficient is nearly zero. Note that the value of transmission coefficient has a jump near the normal incidence; thereafter, as the angle of incidence increases, the value of transmission coefficient increases. At \( \theta = 72^\circ \), the transmission coefficient attains equal value for different values of \( kd \) and thereafter it goes on decreasing towards zero as \( \theta \) approaches \( 90^\circ \). This also shows that at grazing incidence, no transmitted wave take place.

![Figure 3. Variation of transmission coefficient \( T \) versus angle of incidence \( \theta \). Curves 1–4 when \( kd = 0.1, 0.2, 0.3, 0.4 \), respectively; \( \phi = 30^\circ \), \( \varepsilon_\| = 0.5, a_1 = 0.4, a_3 = 0.925 \).](image)

Figure 4 shows the variation of reflection coefficient with normalized wave number \( kd \) for different values of \( \phi \) when \( \theta = 45^\circ \). It is clear that for \( kd = 0.001 \), the reflection coefficient is independent of \( \phi \) and has minimum value. It increases monotonically with the increase of \( kd \). Also as \( \phi \) increases, the reflection coefficient increases for all values of \( kd \) when \( \theta = 45^\circ \).
Fig. 4. Variation of reflection coefficient $R$ versus normalized wave number $kd$ when $\theta = 45^\circ$, $\varepsilon_r = 0.5$. Curves 1–4 when $\phi = 0^\circ$, $30^\circ$, $60^\circ$, $90^\circ$, respectively.

Fig. 5. Variation of transmission coefficient $T$ versus normalized wave number $kd$ when $\theta = 45^\circ$, $\varepsilon_r = 0.5$. Curves 1–4 when $\phi = 0^\circ$, $30^\circ$, $60^\circ$, $90^\circ$, respectively.
Figure 5 shows the variation of transmission coefficient with normalized wave number $kd$ for different values of $\phi$ when $\theta = 45^\circ$. It can be observed that the transmission coefficient decreases monotonically with increasing $kd$ and its value approaches zero as $kd$ approaches 10. However, the transmission coefficient is different for different values of $kd$ at different values of $\phi$.

Figure 6 shows the variation of reflection coefficient with angle $\phi$ for different values of $\theta$. It is clear from figure that at $\phi = 0^\circ$, the reflection coefficient increases as $\theta$ increases through the values $\theta = 30^\circ$, $45^\circ$ and $90^\circ$. For $\theta = 30^\circ$ and $45^\circ$, the value of reflection coefficient starts with 0.30 and 0.71, respectively, and then there is slight decrease in its value with increasing $\phi$. It is also noticed that when $\theta = 90^\circ$, the reflection coefficient first increases and then there is a sudden fall at $\phi = 22^\circ$. Thereafter, it decreases slowly as $\phi$ increases.

Figure 7 shows the variation of transmission coefficient with $\phi$ for different values of angle of incidence. When $\theta = 30^\circ$, $45^\circ$ and $90^\circ$, the value of transmission coefficient remains in the neighbourhood of value 0.53, 0.76 and 0.0, respectively, for all values of $\phi$ running through $0^\circ$ to $90^\circ$. This shows that the transmission coefficient is not much influenced by the parameter $\phi$.

Figure 8 shows the variation of reflection coefficient with angle of incidence for different values of $\varepsilon_H$. We note that the reflection coefficient is influenced most by $\varepsilon_H$ at grazing incidence and its value becomes larger and larger as $\varepsilon_H$ takes higher and higher values.

Fig. 6. Variation of reflection coefficient $R$ versus angle $\phi$. Curves 1, 2, 3 when $\theta = 30^\circ$, $45^\circ$, $90^\circ$, respectively, and $kd = 0.2$, $\varepsilon_H = 0.5$, $a_1 = 0.4$.

Fig. 7. Variation of transmission coefficient $T$ versus angle $\phi$. Curves 1, 2, 3 when $\theta = 30^\circ$, $45^\circ$, $90^\circ$, respectively, and $kd = 0.2$, $\varepsilon_H = 0.5$, $a_1 = 0.4$. 
Fig. 8. Variation of reflection coefficient $R$ versus angle of incidence $\theta$. Curves 1–4 when $\varepsilon_H = 0.05, 0.1, 0.5, 0.9$, respectively, and $kd = 0.2$, $\phi = 30^\circ$, $a_1 = 0.4$.

Fig. 9. Variation of transmission coefficient $T$ versus angle of incidence $\theta$. Curves 1–4 when $\varepsilon_H = 0.05, 0.1, 0.5, 0.9$, respectively, and $kd = 0.2$, $\phi = 30^\circ$, $a_1 = 0.4$. 
Figure 9 shows the variation of transmission coefficient with angle of incidence for different values of $\varepsilon_H$. Here this coefficient increases with increasing value of $\varepsilon_H$.

Fig. 10. Variation of reflection coefficients $R$ versus $\varepsilon_H$. Curves 1, 2, 3 when $(\theta, \phi) = (45^\circ, 30^\circ)$, $(30^\circ, 60^\circ)$, $(45^\circ, 45^\circ)$, respectively.

Fig. 11. Variation of transmission coefficient $T$ versus $\varepsilon_H$. Curves 1, 2, 3 when $(\theta, \phi) = (45^\circ, 30^\circ)$, $(30^\circ, 60^\circ)$, $(45^\circ, 45^\circ)$, respectively.
Its value is found to be independent of $\varepsilon_H$ at grazing incidence, where transmission coefficient vanishes showing that no transmitted wave takes place there.

Figures 10 and 11 shown the variation of reflection and transmission coefficient with $\varepsilon_H$ for different sets of values of $\theta$ and $\phi$. For each set of values, the reflection coefficient shows reverse behaviour than that of transmission coefficient as $\varepsilon_H$ goes through the values 0.0 to 0.2.

6. REMARKS AND CONCLUSIONS

The phenomena of reflection and refraction of seismic plane waves in the layered earth model has been of great importance in geophysics for exploration of various materials beneath the earth’s surface and to understand the earthquake processes. Studies of the past earthquakes equipped with other geophysical data are helpful to find out the location and size of the future earthquake. These estimates are also helpful to earthquake engineers for developing earthquake-resistant structures. Reflection and refraction seismology enables geophysicists to determine gross earth structures, specifically, the crustal and upper mantle structures. Moreover, the resolution obtainable from reflection and refraction seismology is an important method being used by almost all oil exploration companies to map oil reservoirs. Seismic reflection/refraction profiling and wide-angle reflection techniques are of great concern to geophysicists in determining the seismic structure of the oceanic crust. The problem studied here will be helpful in modelling the construction of the reinforced bedding of an earthquake resistant design and dams to control the transmission of earthquake energy to the upper surface in order to avoid the destruction.

In conclusion, a mathematical study of the problem of $SH$ wave propagation through a self-reinforced layer sandwiched between two homogeneous visco-elastic half-spaces is made. Theoretical results indicate that reflection and transmission coefficients depend upon the reinforcement parameters of the sandwiched layer, frequency as well as on the incidence angle. The numerical study reveals that the reflection and transmission coefficients exhibit the reverse behaviour with the increasing values of reinforcement parameters of the sandwiched layer and the normalized wave number. It is found that near the normal incidence both the reflection and transmission coefficients are slightly influenced at small values of $kd$ and $\varepsilon_H$. On the other hand, both the amplitudes ratios are greatly influenced by the increasing values of $kd$ and $\varepsilon_H$.

A c k n o w l e d g e m e n t. One of the authors (Sushil Chaudhary) is grateful to Kurukshetra University, Kurukshetra, for financial support in the form of University Research Scholarship for the completion of this work.
References


Received 16 October 2002
Received corrected version 16 October 2003
Accepted 6 November 2003